Reg. No. | | | | | | | | | |

MT 401

First Semester M.Sc. Degree Examination, December 2018/January 2019
MATHEMATICS
Algebra – I (Repeaters)
Choice Based Credit System – Old Syllabus

Time : 3 Hours Max. Marks : 70

Note : *1) Answer **any five full** questions.*
*2) Answer to **each full** question shall **not** exceed **eight** pages of the answer book. No additional sheets will be provided for answering.*
*3) Use of scientific calculator is **permitted**.*

1. a) Define group and subgroup.
If a, b, c are the elements of a group G then show that
i) $ab = ac \Rightarrow b = c$
ii) $ba = ca \Rightarrow b = c$

b) Prove that in a group
i) The identity element is unique
ii) Every element has unique inverse.

c) Let S be the set of all real numbers except – 1 then prove that S is a group with operation $*$ defined by $a * b = a + b + ab$. **(5+4+5)**

2. a) Prove that the set S of integers n such that $x^n = 1$ is a subgroup of z^+.

b) State and prove Lagrange's theorem.

c) Prove that the homomorphism $\phi : G \to G'$ carries the identity to the identity and inverse to inverse. **(4+6+4)**

3. a) Prove that the subgroup H of a group G is normal if and only if every left coset is also a right coset. Further show that if H is normal then $aH = Ha$ for every $a \in G$.

b) Prove that if N is a normal subgroup of a group G then the product of two cosets aN and bN is again a coset. **(7+7)**

4. a) Prove that every rigid motion is a translation, a rotation, a reflection, a glide reflection or the identity.

   b) Prove that the dihedral group D_n is generated by two elements x, y which satisfy the relations $x^n = 1$, $y^2 = 1$ and $yx = x^{-1}y$. **(7+7)**

5. a) Let H and K be subgroups of a group G. Then prove that the index of $H \cap K$ in H is atmost to the index of K in G.

   b) Prove that the group $GL_2(F_2)$ of invertible matrices with modulo 2 coefficients is isomorphic to the symmetric group S_3. **(7+7)**

6. a) Prove that the center of a p-group has order $p > 1$.

   b) Prove that every group of order p^2 is abelian.

   c) If U be a subset of a group G then prove that the order of the stabilizer stab (U) of U for the operation of left multiplication divides the order of U. **(5+4+5)**

7. a) Prove that there are exactly two isomorphism classes of groups of order 6 which are the classes of cyclic group C_6 and of the dihedrarl group D_3.

   b) Let K be a subgroup of G whose order is divisible by P and let H be a sylow p-subgroup of G. Then show that there is a conjugate subgroup $H' = gHg^{-1}$ such that $K \cap H'$ is a Sylow subgroup of K. **(7+7)**

8. a) Prove that every group of order 15 is cyclic.

   b) Prove that every permutation p not the identity is a product of cyclic permutations which operate on disjoint sets of indices $p : \sigma_1\sigma_2....\sigma_k$ and these cyclic permutations σ_r are uniquely determined by p. **(7+7)**

9. a) Define a ring.

   Let R be a ring with 1 = 0 then prove that R is the zero ring.

   b) Let R denote the ring of continuous real valued functions on $\mathbb{R}^n$. Prove that the map $\phi : \mathbb{R}\ [x_1, x_2, ..., x_n] \to R$ sending a polynomial to its associated polynomial function is an injective homomorphism. **(7+7)**

10. a) If F is a filed then prove that every ideal in the ring F(x) of polynomials in a single variable x is a principal ideal.

    b) Define integral domain and maximal ideal prove that the Kernel of π_1 is either zero or else it is a maximal ideal. **(7+7)**
