Reg. No. | | | | | | | | | |

# MT 404

**First Semester M.Sc. Degree Examination, December 2018/January 2019**
**MATHEMATICS**
**Topology (Repeaters)**
**Choice Based Credit System – Old Syllabus**

Time : 3 Hours Max. Marks : 70

***Note :*** *1) Answer **any five full** questions.*
*2) Answer to **each full** question shall **not** exceed **eight** pages of the answer book. **No** additional sheets will be provided for answering.*
*3) **Use** of scientific calculator is **permitted**.*

1. a) Define a topology on a set X. Define the finite complement topology on a set X and verify that it is a topology on X.

   b) Define a subbasis $\mathcal{S}$ for a topology on a set X and the topology $\mathcal{T}$ generated by $\mathcal{S}$ on X, Prove that $\mathcal{T}$ equals the intersection of all topologies on X that contains S.

   c) Define the standard topology and the lower limit topology on $\mathbb{R}$. Are they comparable ? Justify your answer. **(5+5+4)**

2. a) Define a simple order on a set X and the order topology on a simply ordered set. If $\mathbb{R}$ is the real line in the usual order, prove that a basis for the dictionary order topology on $\mathbb{R} \times \mathbb{R}$ is given by the collection of all open intervals of the form $(a \times b, a \times d)$, where $b < d$.

   b) Let X be an ordered set in the order topology; let Y be a subset of X that is convex in X. Prove that the order topology on Y is the same as the topology Y inherits as a subspace of X.

   c) Let Y be a subspace of a topological space X. Prove that a set A is closed in Y if and only if $A = C \cap Y$ for some set C closed in X. **(5+5+4)**

3. a) For a subset A of a topological space X, define the closure $\overline{A}$ of A. Prove that $x \in \overline{A}$ if and only if every open set U containing x intersects A.

   b) Define a Hausdorff space. Prove that a space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x : x \in X)$ is closed in the product space $X \times X$.

   c) Define the notion of convergence of a sequence in a space X. In a Hausdorff space X, prove that a sequence of point of X converges to at most one point of X. **(5+5+4)**

4. a) Define a continuous map $f : X \to Y$ of a topological space into the other. Prove that the following statements are equivalent for a map $f : X \to Y$.
   i) f is continuous.
   ii) $f(\overline{A}) \subseteq \overline{f(A)}$ for every subset A of X.
   iii) $f^{-1}(C)$ is closed in X whenever C is closed in Y.

   b) Let $f : A \to X \times Y$ given by the equation $f(a) = (f_1(a), f_2(a))$, where A, X, Y are spaces. Then prove that f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. **(9+5)**

5. a) Let $X = A \cup B$, where A and B are closed in X. Let $f : A \to Y$ and $g : B \to Y$ be continuous such that $f(x) = g(x)$ for every $x \in A \cap B$. Prove that f and g can be combined to obtain a continuous function $h : X \to Y$.

   b) Let $\{X_\alpha\}$ be an indexed family of topological spaces; let $A_\alpha \subseteq X_\alpha$ for each $\alpha$. If $\prod_\alpha X_\alpha$ is given either product topology or box topology, then prove that $\overline{\left(\prod_\alpha A_\alpha\right)} = \prod_\alpha \overline{A_\alpha}$.

   c) Let X be a metrizable space and A be a subset of X . If $x \in \overline{A}$, then prove that there is a sequence of points of A converging to x. **(3+7+4)**

6. a) Consider $\mathbb{R}^\omega$, the countably infinite product of $\mathbb{R}$ with itself. Let $f : \mathbb{R} \to \mathbb{R}^\omega$ be a function given by the equation $f(t) = (t, t, t, \ldots)$. Discuss the continuity of f when $\mathbb{R}^\omega$ is given the product topology and also when $\mathbb{R}^\omega$ is given the box topology.

   b) Let $f_n : X \to Y$ be a sequence of continuous functions from the topological space X into the metric space Y. If $(f_n)$ converges uniformly to f , then prove that f is continuous. **(7+7)**

7. a) Define a connected topological space. Prove that the image of a connected space under a continuous map is connected.

   b) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

   c) Prove that the finite product of connected spaces is connected. **(4+5+5)**

8. a) Define a linear continuum. If L is a linear continuum in the order topology, then prove that L is connected.

   b) If X is a simply ordered set having the least upper bound property, then prove that each closed interval in X is compact in the order topology. **(7+7)**

9. a) Prove that every compact subspace of a Hausdorff space is closed.

   b) Let X be a space and Y be a compact space. If N is an open subset of the product space $X \times Y$ containing the slice $x_0 \times Y$ of $X \times Y$, then prove that N contains some tube $W \times Y$ about $x_0 \times Y$, where W is a neighbourhood of $x_0$ in X .

   c) Show that a finite union of compact subspaces of X is compact. **(6+4+4)**

10. a) Define a limit point compact space. Prove that every compact space is limit point compact.

    b) Define a sequentially compact space. If a metrizable space is sequentially compact, then prove that it is compact. **(5+9)**

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