Reg. No. | | | | | | | | | |

MT 503

Third Semester M.Sc. Degree Examination, December 2018/January 2019
MATHEMATICS (Repeaters)
Ordinary Differential Equations
Choice Based Credit System – Old Syllabus

Time : 3 Hours Max. Marks : 70

Note *: 1) Answer* ***any five full*** *questions.*
2) Answer to ***each full*** *question shall* ***not*** *exceed* ***eight*** *pages of the answer book.* ***No*** *additional sheets will be provided for answering.*
3) Use of scientific calculator is ***permitted****.*

1. a) If $x_1(t), x_2(t), ..., x_n(t)$ are solutions of $L_n(x) = 0$ then prove they are linearly independent if and only if $w[x_1, x_2,...,x_n](t) \neq 0, \forall t \in I$.

   b) Compute the Wronskian of t^2 and $t\,|t|$ on R and show that they are linearly independent on R.

   c) Solve $x'' + x' = \sin t$. **(7+4+3)**

2. a) If Φ_1 is a solution of $a_0(t)x'' + a_1(t)x' + a_2(t)x = 0$ on I and if $\Phi_1(t) \neq 0, \forall t \in I$.

   Show that $\Phi_2(t) = \Phi_1(t)\int_{t_0}^{t} \frac{1}{(\phi_1(s))^2} \exp\left[\int_{t_0}^{s} \frac{-a_1(\xi)}{a_0(\xi)} d\xi\right] ds$ is another solution.

   Further show that $\Phi_2(1)$ and $\Phi_2(t)$ are linearly independent on I.

   b) Describe the method of variation of parameters to find the general solution of $L_n(x) = b(t)$. **(7+7)**

3. a) Obtain the series solution of Legendre equation $(1 - t^2)\,x'' - 2tx' + p(p + 1)\,x = 0$.

   b) State and prove orthogonality property for Legendre polynomials. **(7+7)**

4. a) Obtain the series solution of $t^2 x'' - tx' + (t^2 - \alpha^2)\,x = 0, t > 0$. Where α is the constant.

   b) State and prove the orthogonality of Bessel's polynomial. **(7+7)**

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5. a) Find the solution of Legendre differential equation $(1 - x^2)\, y'' - 2xy' + \alpha(\alpha + 1)y = 0$.

   b) Prove that $\left[J_{\frac{1}{2}}(t)\right]^2 + \left[J_{-\frac{1}{2}}(t)\right]^2 = \frac{2}{\pi t},\ t > 0.$ **(7+7)**

6. a) Let A(t) be a $n \times n$ continuous matrix function defined on a closed and bounded interval I. Then show that those exists a solution to the IVP $x' = A(t)x$, $x(t_0) = x_0$ where $t, t_0 \in$ I. Further show that this solution is unique.

   b) Find the fundamental matrix for $x' = A(t)x$ where $A = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$. **(8+6)**

7. a) Find the first four Picard's approximation of the initial value problem $x' = t + x$, $x(0) = 1$. Further find the limit of these approximations.

   b) Let A(t) be an $n \times n$ continuous matrix defined on $(-\infty, \infty)$ and periodic with period w. If $\phi(t)$ is a fundamental matrix of $x' = A(t)x$, then prove that $\phi(t + w)$ is also a fundamental matrix. Justify that for any such $\phi(t)$, there exists a periodic nonsingular matrix p(t) with period w and a constant matrix R such that $\phi(t) = P(t)e^{tR}$. **(7+7)**

8. a) If A is an $n \times n$ constant matrix, find a fundamental matrix of $x' = A(t)x$, by considering the cases where the eigenvalues of A are all distinct.

   b) State Lipschitz condition with respect to x for a function f(t, x) defined on domain D on R^2. Give sufficient condition for f(t, x) to satisfy a Lipschitz condition. Discuss the necessary part for f(t, x) to satisfy a Lipschitz condition. **(7+7)**

9. a) Solve $x'' + 4x = t$, $0 \le t \le 3$, subject to the boundary conditions $x(0) = 0$, $x'(3) = 0$ by determining the Green's function.

   b) State Sturm-Liouville boundary value problem with separated or periodic boundary conditions. Find the eigenvalues and the associated eigen functions of the Sturm-Liouville problem : $x' + \lambda x = 0$, $0 < t < L$, $x'(0) = 0$, $hx(L) + x'(L) = 0$ $h > 0$. **(7+7)**

10. a) Expand the function $f(t) = \pi t - t^2$, $0 \le t \le \pi$ in terms of the eigenfunctions of Sturm-Liouville problem : $x'' + \lambda x = 0.x(0) = x(\pi) = 0$.

    b) Show that the eigenvalues of a Sturm-Liouville boundary value problem are real. **(7+7)**

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