Reg. No. | | | | | | | | | |

MTE 501

Third Semester M.Sc. Degree Examination, Dec. 2018/Jan. 2019
MATHEMATICS
Differential Equations and Applications
(Choice Based Credit System – New Syllabus) (Open Elective)

Time : 3 Hours Max. Marks : 70

Note : *1) Answer **any five full** questions.*
*2) Answer to **each full** question shall **not** exceed **eight** pages of the answer book. **No** additional sheets will be provided for answering.*
*3) **Use** of scientific calculator is **permitted**.*

1. a) If k is a given nonzero constant, show that the function $y = ce^{kx}$ are only the solutions of the differential equation $\frac{dy}{dx} = ky$.

b) A tank contains 50 gallons of brine in which 75 pounds of salt are dissolved. Beginning at time t = 0, brine containing 3 pounds of salt per gallon flows in at the rate of 2 gallons per minute and the mixture flows out at the same rate. When will there be 125 pounds of dissolved salt in the tank? How much dissolved salt is in the tank after a long time ?

c) A bacterial culture of population x is known to have a growth rate proportional to x itself. Between 6 P.M. and 7 P.M. the population triples. At what time will the population become 100 times what it was at 6 P.M. ? **(2+7+5)**

2. a) Uranium - 238 decays at a rate proportional to the amount present. If x1 and x2 grams are present at times t_1 and t_2, show that the half-life is $\frac{(t_2 - t_1)\log 2}{\log\left(x_1/x_2\right)}$.

b) A pot of carrot-and-garlic soup cooling in air at 0° C was initially boiling at 100° C and cooled 20° during the first 30 minutes. How much will it cool during the next 30 minutes ?

c) If the half-life of a radioactive substance is 20 days, how long will it take for 99 percent of the substance to decay ? **(5+5+4)**

P.T.O.

3. a) If the air resistance acting on a falling body of mass m exerts a retarding force proportional to the square of the velocity of the falling body, then the differential equation becomes $\frac{dv}{dt} = g - cv^2$, where $c = k/m$. If v = 0 when t = 0, find v as a function of t, what is the terminal velocity in this case ?

b) If a body falls from rest at y = 0, with usual notations derive that $v = \sqrt{2gy}$.

c) Derive second order differential equation in the form $\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$, where l is the length of the pendulum and θ is the angular displacement at any time t. Hence find expression for the period T. **(5+4+5)**

4. Discuss undamped and damped harmonic vibrations of a cart attached to a wall by means of a spring. Derive the expressions for the period and frequency of oscillation in each case. **14**

5. a) Find

i) $L\{x^2 \sin ax)$

ii) $L\{x^{3/2} + x^{10} e^x\}$

iii) $L\{e^{3x} \cos 2x + (1 - x^2)e^{-x}\}$.

b) Using Laplace transforms evaluate the integral $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} dx$ where a and b are positive constants.

c) Find $L^{-1}\left\{\frac{p+3}{p^2+2p+5}\right\}$. **(8+3+3)**

6. a) Using Laplace transforms, solve the initial value problem $y'' + 5y' + 6y = 5e^{3t}$, $y(0) = y'(0) = 0$.

b) Obtain the power series solution of the Legendre equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$, where p is real constant. **(7+7)**

7. a) State and prove orthogonal property of Legendre polynomials $P_n(x)$.

b) Obtain the power series solution of n^{th} order Bessel equation

$x^2y'' + xy' + (x^2 - p^2)y = 0$, where p is non-negative constant. **(7+7)**

8. a) Establish the orthogonal property of the Bessel functions.

b) If $H_n(x)$ denote Hermite polynomial of degree n, prove that

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n .$$ **(7+7)**