Reg. No. | | | | | | | | | |

STS 455

II Semester M.Sc. Degree Examination, September/October 2022
STATISTICS
Actuarial Statistics

Time : 3 Hours   Max. Marks : 70

Note : *1) Question No. **1** is **compulsory**.*
*2) Answer **any four** questions from the remaining **seven** questions.*

1. Answer **any six** questions. **(6×3=18)**
   a) Give an economic justification for insurance system with example. Also discuss about its limitations.
   b) Define distribution function and survival function of T(x). Derive these in terms of survival function.
   c) Derive an expression for curtate expectation life.
   d) What is select life table ? When it is used ?
   e) Suppose that Gompertz' law applies with $\mu_{30} = 0.000130$ and $\mu_{50} = 0.000344$. Calculate ${}_{10}p_{40}$.
   f) Differentiate between n-year pure endowment insurance and n-year endowment insurance.
   g) Explain annuity certain and annuity due and give their expressions.
   h) Describe fully continuous premiums, fully discrete premiums.

2. a) Derive $S_x(t)$ and force of mortality under Makeham's law of mortality.
   b) Suppose the survival function of life length random variable is $S(x) = 1 - x^2/100$ for $0 < x \le 10$. Find
      i) density function of life length random variable;
      ii) density function of T(x)
      iii) ${}_3p_3$
      iv) complete expectation of life.
   c) Let the probability distribution function of length of life be $F_0(x) = 1 - (1 - x/120)^{1/6}$ for $0 < x \le 120$, calculate e_x^0. **(4+6+3)**

P.T.O.

3. a) Show that the density of T(x) can be written $f_{Tx}\ t = {}_tp_x\mu_{x+t}$ and also for $\mu_{x+t} = t$ for $t \geq 0$. Calculate ${}_tp_x\ \mu_{x+t}$ and e_x^0.

   b) Define time until death for a person age x. Let $F_0\ t = 1 - (1 - t/120)^{1/6}$, for $0 \leq t \leq 120$. Calculate the probability that

      i) a life aged 30 dies before age 50 and

      ii) a life aged 40 survives beyond age 65.

   c) A life aged (40) is subject to an extra risk for the next year only. Suppose the normal probability of death is 0.004, and that the extra risk may be expressed by adding the function 0.03 (1 – t) to the normal force of mortality for this year. What is the probability of survival to age 41 ?

(5+5+3)

4. a) What are the fractional age assumptions ? Show that under the assumption of uniform distribution of deaths in the year of death that K(x) and T(x) – K(x) are independent and that T(x) – K(x) has the uniform distribution on the interval (0, 1). Prove the equivalence of UDD1 and UDD2.

   b) The Gompertz law of mortality is defined by the requirement that $\mu_t = Ac^t$ for some constants A and c. What restrictions are there on A and c for this to be a force of mortality ? Write an expression for ${}_tp_x$ under Gompertz law.

   c) Given that q70 = 0.01422 and q71 = 0.01310, calculate, 0.7 q 70.6 assuming a uniform distribution of deaths.

(6+5+2)

5. a) Part of a select life table with two year selection period is given below.

[x]	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	x + 2
30	9907	9905	9901	32
31	9903	9901	9897	33
32	9899	9896	9893	34

   Calculate (i) 2p[32], (ii) 2q[30]+1, (iii) 2|q [31], (iv) 2q32, (v) 2|2q[30].

   b) If l_0 is 1,00,000 and $S(x) = 1/(1 + x^2)$, calculate l_x by random survivorship method for x = 15.

   c) For an amount A invested today one gets Rs. 3,00,000 after six years with interest compounded quarterly with nominal interest rate $i^{(4)} = 0.08$. What is A ?

(5+5+3)

6. a) Define n-year endowment insurance. Show that $\bar{A}_{x:\overline{n}|} = \bar{A}^{1}_{x:\overline{n}|} + \bar{A}_{x:\frac{1}{\overline{n}|}}$.

b) Define deferred insurance. Prove that $_{u|}\bar{A}^{1}_{x:\overline{n}|} = \bar{A}^{1}_{x:\overline{u+n}|} - \bar{A}^{1}_{x:\overline{u}|}$.

c) Consider an insurance policy issued to (x) under which the death benefit is $(1 + j)^t$ if death occurs at age x + t, with the death benefit being payable immediately on death.

i) Derive an expression for the actuarial present value of the death benefit if the policy is an n-year term insurance.

ii) Derive an expression for the actuarial present value of the death benefit if the policy is a whole life insurance. **(4+5+4)**

7. a) Describe in words the benefits with the present values given and write down an expression in terms of actuarial functions for the expected present value.

$$Y_1 = \begin{cases} \bar{a}_{\overline{T_x}|} & \text{if } T_x \le 15, \\ \bar{a}_{\overline{15}|} & \text{if } T_x > 15. \end{cases}$$

and $Y_2 = \begin{cases} a_{\overline{15}|} & \text{if } 0 < K_x \le 15, \\ a_{\overline{K_x}|} & \text{if } K_x > 15 \end{cases}$.

b) An annuity immediate pays an initial benefit of one per year, increasing by 10.25% every four years. The annuity is payable for 40 years. If the effective interest rate is 5% find an expression for the present value of this annuity.

c) Explaining the terms involved, show that $\sum_{k=0}^{\infty} \ddot{a}_{k+1} + 1_k q_x = \sum_{k=0}^{\infty} v^k {}_k p_x$.

(5+5+3)

8. a) An insurer issues a 25-year annual premium endowment insurance with sum insured \$100000 to a select life aged 30. The insurer incurs initial expenses of \$ 2000 plus 50% of the first premium and renewal expenses of 2.5% of each subsequent premium. The death benefit is payable immediately on death. (i) Write down the gross future loss random variable. (ii) Calculate the gross premium using the standard select survival model with 5% per year interest.

b) If the life length random variable X has a uniform distributed over (0, 120), determine the actuarial value of the benefit paid under continuous case. Also, determine the variance of present value of the benefit. **(8+5)**

Reg. No. [ ]

STS 557

Fourth Semester M.Sc. Degree Examination, September/October 2022

STATISTICS

Data Mining Techniques

Time : 3 Hours

Max. Marks : 70

Note : *Question number* ***1*** *is* ***compulsory.***

Answer ***any four*** *questions from the remaining seven questions.*

1. Answer **any six** questions. **(6×3=18)**

   a) Discuss any two applications of data mining in industry.

   b) Which are the tools used for data integration in data mining ?

   c) Discuss various kind of data used in data mining.

   d) Differentiate between Online Transaction Processing (OLTP) and Online Analytical Processing (OLAP).

   e) Explain CART approach of construction of decision trees.

   f) What is the difference between supervised learning and unsupervised learning ?

   g) Mention any three "similarity and distance measures" and its characteristics.

   h) Explain with examples crossover and mutation with reference to genetic algorithm.

2. a) Describe the steps involved in data mining when viewed as a process of knowledge discovery.

   b) Explain the different schemes of multidimensional data modelling with examples. **(7+6)**

3. a) Taking an example of applications of artificial intelligence in data mining, describe the method and use of the same.

   b) Explain with the help of an example, the different kinds of OLAP operations performed in a data cube. **(7+6)**

P.T.O.

4. a) Explain data integration and data transformation of data pre-processing methods.

   b) Explain the steps involved in Andrews plots and explain Chernoff faces. Write down the difference among them. (7+6)

5. a) Explain the principle of ID3 algorithm and using an application describe the method.

   b) Using an example describe decision tree problems. How the classifications are carried out ? (6+7)

6. a) Explain in detail the regression based classification.

   b) Explain the working theory of k-means clustering method. (7+6)

7. a) Discuss partitioning algorithm and its applications.

   b) How Does DBSCAN quantify the neighbourhood of an object and how clusters are formed ? (7+6)

8. a) Explain how Bootstrap method is used to generate an empirical estimate of the sampling distribution of an estimate.

   b) Explain Gibbs sampler. Further explain how it is used in Markov Chain Monte Carlo. (6+7)

Reg. No. | | | | | | | | | |

STH 551

IV Semester M.Sc. Degree Examination, September/October 2022
STATISTICS
Design and Analysis of Experiments

Time : 3 Hours Max. Marks : 70

Note : *Question No. 1 is* ***compulsory****. Answer* ***any four*** *questions from the remaining seven questions.*

1. Answer **any six** sub-divisions. **(6×3=18)**

   a) Obtain the least square estimator of β in the linear model. Is the least square estimator of β unbiased ?

   b) When do you say that a linear parametric function is estimable ? Give an example for (i) an estimable parametric function (ii) non-estimable parametric function.

   c) Describe the general block design.

   d) Define the information matrix C. Prove that matrix C is symmetric.

   e) Obtain the efficiency factor of a Balanced Incomplete Block Design (BIBD).

   f) Describe Duncan's multiple comparison test.

   g) In a Randomised Block Design (RBD) with single missing observation, obtain the variance of the estimated value.

   h) Giving illustrations, distinguish between complete and partial confounding in factorial experiments.

2. a) State and prove Gauss Markov theorem.

   b) Define general linear model, by stating the assumptions. Derive the test procedure to test $H_0 : K' \beta = m$. **(6+7)**

3. a) In a general block design, state and prove a necessary and sufficient conditions for a linear parametric function $a'\theta$ to be estimable.

   b) Write down the normal equations of a general block design and give a solution to it.

   c) Prove that in a connected design, every treatment contrast is estimable. **(6+4+3)**

P.T.O.

4. a) Show that an RBD is connected and variance balanced.
   b) Show that, treatment contrasts and block contrasts are orthogonal in RBD.
   c) Describe Tukey's test for non-additivity. (4+4+5)

5. a) State a BIBD model. Show that it is a variance balanced design.
   b) Obtain the intra block analysis of a BIBD. (4+9)

6. a) State the one-way classification model with single covariate. Stating the assumptions, derive the likelihood ratio test procedure to test the significance of the treatment effects.
   b) Discuss the estimation of single missing value in an RBD and outline an approximate test to test the relevant hypothesis. (8+5)

7. a) What are main effects and interaction effects in a 2^3 factorial experiment ?
   b) Describe Yates technique to compute the sum of squares due to the main effects and the interaction effects in a 2^3 factorial experiment.
   c) Briefly indicate the analysis in a 2^3 factorial experiment in which the effect AB is confounded in replicate I, AC in replicate II and BC in replicate III. (4+4+5)

8. a) Describe Youden Square Design (YSD). Obtain the normal equations of YSD and hence derive the best estimator for the model parameters.
   b) Explain the importance of ANCOVA and derive the test procedure for ANCOVA with two-way classification. (6+7)

Reg. No.

STH 452

II Semester M.Sc. Degree Examination, September/October 2022
STATISTICS
Distribution Theory

Time : 3 Hours Max. Marks : 70

Note : *Question No. **1** is **compulsory**. Answer **any four** questions from the remaining seven questions.*

1. Answer **any six** sub-divisions. **Each** question carries **3** marks. **(6×3=18)**

   a) Find the m.g.f. of normal random variable with parameters μ and σ^2.

   b) If X and Y have joint pdf $f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$. Examine whether X and Y are independent.

   c) Define non-central t distribution. State its mean and variance.

   d) Prove that, $V(X) = V(E(X|Y)) + E(V(X|Y))$.

   e) Let $X \sim U(0, 1)$. Find the transformation $Y = g(X)$ which has the density function $f_y(y) = e^{-y}, y > 0$.

   f) Prove or disprove : Exponential distribution satisfies lack-of-memory property.

   g) Define Wishart distribution and mention any one of its properties.

   h) Let $Y_1, Y_2, \ldots, Y_n$ are order statistics of a random sample from an exponential distribution with parameter λ. Find the joint distribution of Y_1, Y_n.

Answer **any four full** questions from the following, **each** question carries **13** marks. **(4×13=52)**

2. a) Obtain p.g.f. of a negative binomial distribution, show that negative binomial is a members of power series distribution.

   b) State and establish any two properties of Weibull distribution. **(7+6)**

3. a) If X is any continuous random variable with cdf F. Find the distribution of $Y = F(X)$.

   b) Define truncated distribution. Write down the probability mass function of a Poisson distribution truncated at zero and find its mean and variance.

   c) What is probability integral transformation ? What is its use ? **(5+5+3)**

4. a) Let X and Y be iid gamma random variables. Find the joint distribution of $U = \frac{X}{X+Y}$ and $V = (X + Y)$. Obtain the conditional distribution of V given U = u.

   b) Show that a random variable X has standard Cauchy distribution if and only if $\frac{1}{X}$ also has standard Cauchy distribution.

   c) Define extreme value distribution and state its important properties. **(6+4+3)**

5. a) Prove or disprove : Mean and variance of a random sample of a normal distribution are independent.

   b) Define logistic distribution. Obtain its moment generating function. **(8+5)**

6. a) Let $Y = (Y_1, Y_2, \ldots, Y_n)'$ where Y_i's are iid N(0, 1) random variables. Prove that a necessary and sufficient condition that $Y'AY$ has chi-square distribution is that A is idempotent.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential with parameter θ. Let $Y_1, Y_2, \ldots, Y_n$ be order statistics. Then show that $Y_s - Y_r$ and Y_r are independent. **(5+8)**

7. a) Derive the characteristic function of Wishart distribution.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Show that $\sum_{i=1}^{n}(X_i - \bar{x})^2 / \sigma^2$ has χ^2-distribution with n-2 degrees of freedom. **(7+6)**

8. a) Define multivariate normal distribution. Find the distribution of linear combinations of components of a vector having multivariate normal distribution.

   b) Prove or disprove : The conditional distribution of any sub-vector given the other components in a multivariate normal random vector is again multivariate normal. **(7+6)**

Reg. No. | | | | | | | | |

STH 454

II Semester M.Sc. Degree Examination, Sept./Oct. 2022
STATISTICS
Econometrics

Time : 3 Hours Max. Marks : 70

Instructions : *Question No.* ***1*** *is* ***compulsory****. Answer* ***any four*** *from the remaining* ***seven*** *questions.*

1. Answer **any six** subdivisions from the following : **(6×3=18)**
 a) Define econometrics. Briefly explain the various aspects we come across in econometrics.
 b) Show that $\hat{\beta}_{ols}$ and $\hat{\sigma}^2$ are independently distributed.
 c) Define Mallow's C_p statistic. Explain its role in the best subset selection.
 d) Obtain the restricted least squares estimators of β in $Y = X\beta + \varepsilon$ under the restriction $R\beta = r$ and show that it dominates the OLS estimator.
 e) Describe multicollinearity in multiple linear regression model. Explain its consequences on OLSE.
 f) Explain stochastic regression model. Mention its consequences on least squares estimate of parameters.
 g) Explain Durbin-Watson test for detecting autocorrelation in regression model.
 h) Define the following in the context of system of simultaneous equation.
  i) Over identification and exact identification.
  ii) Zero restrictions.

2. a) Obtain the least squares estimators of slope and intercept terms along with their standard errors in a simple linear regression model.
 b) Define recursive residuals. Explain the procedure to obtain the same.
 c) Define Multiple Linear Regression Model. By stating basic ideal conditions, derive the OLS estimator of model parameters. **(5+4+4)**

P.T.O.

3. a) Explain logistic regression model. Describe a test for testing significance of the Individual regressor in model.
   b) Show that for the model satisfying all the basic ideal conditions, OLS estimators of β and σ^2 are jointly sufficient for β and σ^2.
   c) Explain the concept of outlier analysis. (4+6+3)

4. a) Derive the maximum likelihood ratio test statistic to test 'm' exact linear restrictions on the regression coefficient vector.
   b) Explain asymptotically uncooperative regressors. Propose a consistent estimator for regression coefficient. (7+6)

5. a) Define Best Linear Unbiased Predictor (BLUP). Obtain the expression for BLUP in simple linear regression model satisfying all the basic ideal conditions.
   b) Show that in case of non-zero mean error term in the regression model, simultaneously unbiased estimators for β and σ^2 cannot be obtained.
   c) Explain a test procedure for detecting heteroscedasticity. (5+5+3)

6. a) Describe the Instrumental Variable (IV) method of estimation and the properties of the resulting estimators.
   b) Derive Generalised Least square estimator of regression coefficients in heteroscedastic model. (7+6)

7. a) Explain errors in variables in regression model. Show that the OLSE of regression coefficients are inconsistent in this case.
   b) Explain the estimation of the parameters of auto correlated regression model. Let

   $Q = \alpha P + \beta Z + U1$

   $P = \gamma Q + U2$

   where Q denotes the quantity, P denotes the price, Z denotes personal income. Assume that Q and P are endogenous variables. Examine whether the above system is identifiable.
   c) Distinguish between structural and reduced form of system of simultaneous equations. (3+5+5)

8. a) Explain grouping of equations. Explain how to estimate the regression coefficient in this case.
   b) Show that OLS estimators of reduced form parameters are consistent.
   c) Explain Two Stage Least Squares estimation in simultaneous equation model. State its properties. (3+5+5)

Reg. No. | | | | | | | | | |

STS 554

IV Semester M.Sc. Degree Examination, September/October 2022
STATISTICS
Financial Time Series

Time : 3 Hours Max. Marks : 70

Instructions : *1) Question No. **1** is **compulsory**.*
*2) Answer **any four** questions from the **remaining**.*

1. Answer **any six** subdivisions from the following. **(6×3=18)**

a) Explain the special features of financial time series. How it is different from classical time series ?

b) Explain the terms :

i) Continuously compounded multiperiod return

ii) Portfolio return

iii) Dividend payment.

c) Define sample skewness and sample kurtosis of the return. Give the test statistics for

i) testing skewness of return is zero

ii) testing excess kurtosis of return is zero.

d) Define sample autocorrelation function (ACF) of a stationary time series. Obtain autocorrelation function of the time series $X_t = 0.9X_{t-1} + \varepsilon_t$. Is it stationary ? Justify.

e) Explain the test procedure for detecting unit root in a time series.

f) Describe seasonal integrated autoregressive moving average model.

g) Define volatility and state its properties.

h) Explain co integration and error correction models.

2. a) Define ARCH(p) model. Explain a test procedure for testing the ARCH effect.

b) Let Y_t follows ARCH(1) process. Show that $\{Y_t\}$ is uncorrelated. Obtain ACF of Y_t^2. Show that marginal distribution of $\{Y_t\}$ is heavy tailed.

c) Obtain the Yule-Walker equation for the ARCH(p) process. **(4+5+4)**

P.T.O.

3. a) Define moving average process of order q. Obtain its variance and auto covariance function.

b) Let X_t follows AR(1) and $\bar{X}_n = \frac{\sum_{t=1}^{n} X_t}{n}$ find $Var(\bar{X}_n)$.

c) Suppose that the daily log return of a security follows the model rt = 0.01 + 0.2rt – 2 + at, where {at} is a Gaussian white noise series with mean zero and variance 0.02. What are the mean and variance of the return series rt ? Compute the lag-1 and lag-2 autocorrelations of rt. **(5+3+5)**

4. a) Explain Yule-Walker method of estimation for an AR(p) model.

b) Write the model in backward shift operator $X_t = 1.5X_{t-1} - 0.6X_{t-2} + \varepsilon_t$. Examine for stationary. Obtain the Yule-Walker equations for this model and solve these equations to obtain ρ_1 and ρ_2. **(5+8)**

5. a) Define GARCH (p, q) model for the return series. Obtain the variance and kurtosis of return series which follows GARCH(1 1).

b) Obtain the maximum likelihood estimates of parameters of ARCH(1) process. **(7+6)**

6. a) Define Exponential GARCH and GARCH in mean models. State elementary properties of these models.

b) Explain how GARCH(1 1) is related to ARMA(1 1) process. Whether writing GARCH as ARMA solve the problem of estimation. Explain.

c) Explain the order determination procedure of classical financial time series models. **(5+4+4)**

7. a) Explain residual analysis in time series modeling. Explain the related tests based on residuals.

b) Obtain h-step ahead forecast of GARCH(1, 2) process.

c) Explain the steps involved in building a financial time series model. **(4+5+4)**

8. a) Obtain the autocorrelation function of GARCH(1 1) process.

b) Obtain Explicit expression for ACF of ARMA(1 1) process.

c) Derive the L-step ahead forecast equation of ARCH (p) process. **(4+4+5)**

Reg. No. | | | | | | | | |

STH 402

First Semester M.Sc. Degree Examination, May 2022
STATISTICS
Matrix Theory and R-Programming

Time : 3 Hours Max. Marks : 70

Note : *Question No. 1 is* ***compulsory.*** *Answer* ***any four*** *questions from the remaining seven questions.*

1. Answer **any six** sub divisions. **(3×6=18)**
   a) Show that inverse of matrix is unique if it exists.
   b) Prove or disprove : Intersection of two subspaces is also a subspace.
   c) Define idempotent matrix. Obtain its Eigen values.
   d) Define characteristic equation and characteristic root of a matrix.
   e) Show that a square matrix A is nilpotent if and only if all of its Eigen values are zero.
   f) What are the advantages of R ?
   g) Mention the rules to be followed while defining variable names in R.
   h) Write a note on logical operators in R.

2. a) Define linear dependence and independence of a set of vectors.
   Examine whether the vectors $X_1 = (3, 2, 1)$, $X_2 = (1, 2, 3)$, $X_3 = (2, 3, 8)$ and $X_4 = (1, 1, 1)$ are linearly dependent or not.
   b) Prove that the number of vectors in any basis of a vector space is same as in any other basis. **(7+6)**

3. a) If A and B are n-rowed squared matrices prove that,
   $\text{Rank}(AB) \geq \text{Rank}(A) + \text{Rank}(B) - n$.
   b) Show that a set of non-null mutually orthogonal vectors is necessarily a linear independent set.
   c) Show that the characteristic vectors corresponding to distinct characteristic roots of a matrix are linearly independent. **(6+4+3)**

P.T.O.

4. a) State and prove rank nullity theorem.

   b) Show that Geometric Multiplicity of a characteristic root cannot exceed its Algebraic Multiplicity. **(7+6)**

5. a) Define g-inverse of a matrix. Find the same for $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 5 \\ 1 & 1 & 3 \end{bmatrix}$.

   b) Define rank, index and signature of quadratic form. Find rank and index of the quadratic form $x_1^2 - 2x_2^2 + 3x_3^2 + 4x_2x_3 + 6x_3x_1$. **(7+6)**

6. a) What are the different data structures in R ? Explain about them.

   b) Write about 'for' and 'repeat' loops in R. **(8+5)**

7. a) Write a note on Packages in R and explain any two methods of package installation in R.

   b) Write a note on handling missing values in R.

   c) Explain basic structure of user defined functions in R. Give an example. **(4+3+6)**

8. a) Explain any three statistical tests available in R.

   b) Explain graphical facilities in R.

      i) High level plotting commands.

      ii) Low level plotting commands.

   c) Write R code to compute Eigen values and Eigen vector for matrix

   $A = \begin{bmatrix} 6 & 2 & 3 \\ 9 & 6 & -6 \\ -2 & 8 & 1 \end{bmatrix}$. **(3+6+4)**

Reg. No. | | | | | | | | | |

STS 505

Third Semester M.Sc. Examination, April/May 2022
STATISTICS
Multivariate Analysis

Time : 3 Hours

Max. Marks : 70

Instruction : *Question No. **1** is **compulsory**. Answer **any four** questions from the remaining **seven** questions.*

1. Answer any **six** sub divisions. **(3×6=18)**

a) If $X \sim N_p(\mu, \Sigma)$ where Σ is positive definite matrix. Then obtain the distribution of $Y = DX$ where 'D' is a non-singular matrix.

b) Explain the invariance property of Hoteling's T^2 – statistic.

c) Describe the construction and importance of the chi-square plot.

d) If X is a random vector with dispersion matrix $\begin{bmatrix} 1 & 2 \\ 2 & 10 \end{bmatrix}$, obtain the first principal component.

e) Distinguish between classification and discrimination with illustrative examples.

f) Explain the procedure of estimating the total probability of misclassification in discriminant analysis.

g) What is factor analysis ? Give an illustrative example.

h) Describe the steps of the k-means algorithm for clustering.

2. a) Let $X_1, X_2, \ldots X_n^-$ are random samples from $N_p(\mu, \Sigma)$, obtain the maximum likelihood estimator of μ and Σ.

b) With usual notation prove that sample mean vector and sample covariance matrix are independently distributed. **(7+6)**

3. a) Define Mahalanobis D^2 statistic. Mention its application.

b) Based on a sample of independent observations from $N_p(\mu, \Sigma)$, derive the likelihood ratio test for testing $H_1 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. Also, state distribution of the test statistic under the null hypothesis. **(4+9)**

4. a) Define Hoteling's T^2 – Statistic and state its applications. Explain any one of them in detail.

   b) Derive the expression for the first and second Principal Components based on the population covariance matrix Σ.

   c) Explain the problem of Scaling in the Principal Component Analysis. (4+6+3)

5. a) Define canonical correlations and canonical variates. Derive the expression for first set of canonical variates, given the population covariance matrix Σ.

   b) Derive the classification rules to classify an observation into one of the two multivariate normal populations with equal covariance matrices. (8+5)

6. a) Explain the idea of projection involved in constructing Fisher's linear discriminant function. Derive Fisher's linear discriminant rule to classify the observations.

   b) Describe the test for the validity of linear discriminant function. (8+5)

7. a) Given two populations Π_1 and Π_2, derive the classification rule based on the expected cost of misclassification (ECM).

   b) Stating the assumptions, explain an orthogonal factor model. Also, explain the commonalities and specific variances.

   c) Explain factor scores and factor rotations. (4+5+4)

8. a) Explain the principal component method of estimating the factor loadings.

   b) Explain Ward's hierarchical clustering procedure.

   c) Explain the Agglomerative clustering algorithm methods. Also, explain the use of the linkage methods in the clustering algorithm. (5+3+5)

Reg. No. | | | | | | | | |

STS 552

Fourth Semester M.Sc. Degree Examination, September/October 2022

STATISTICS

Operations Research

Time : 3 Hours Max. Marks : 70

Notes : *1) Question No.* ***1*** *is* ***compulsory****.*
2) Answer ***any four*** *questions from the remaining* ***seven*** *questions.*

1. Answer **any six** sub-divisions. **(3×6=18)**

a) Define the following terms :

i) Basic solution

ii) Degenerate basic feasible solution

iii) Hyper-plane.

b) Write a note on Big-M method of solving Linear Programming Problem (LPP).

c) What are the steps involved in writing a primal LPP into its dual form ?

d) Prove that if primal variable is unrestricted in sign then associated dual constraint is an equation.

e) Explain the shortest distance problem.

f) Explain the characteristics of dynamic programming problem.

g) Distinguish between (Q, r) and (S, s) polices.

h) Describe periodic review in inventory modeling.

2. a) Prove that the linear objective function of an LPP attains its optimal (minimum) value at an extreme point of the convex feasible region.

b) Explain two phase method of solving LPP. **(8+5)**

3. a) Illustrate the steps involved in formulating a LPP with an example.

b) Explain graphical method of solving LPP with different cases.

c) Describe the condition for selecting a non basic variable while solving a standard LPP. **(5+4+4)**

4. a) State and prove the complementary slackness theorem.
   b) Prove that in a primal-dual pair of linear programming problem's, if z(x) and z(w) be the primal and dual objective functions respectively and $\bar{x}$ and $\bar{w}$ are the pair of primal and dual feasible solution with $z(\bar{x}) = z(\bar{w})$. Then $\bar{x}$ and $\bar{w}$ is an optimal solution pair of the primal and dual LP.
   c) Explain the applications of duality theory. **(5+4+4)**
5. a) State and prove weak duality theorem.
   b) Explain dual simplex method of solving a linear programming problem.
   c) Explain Gomory's method of generating a cut. **(5+4+4)**
6. a) Obtain the optimum values for ordered quantity and for shortages when the inventory system allows decay.
   b) Explain the motives for holding inventory. **(8+5)**
7. a) Explain a heuristic solution procedure of single period model under simple (Q, r) system.
   b) Derive the Wilson-Harris policy.
   c) Write a note on stochastic inventory models. Explain the different approaches to solve such system. **(5+4+4)**
8. a) Define the term "Sensitivity Analysis". Explain its different cases.
   b) Write a note on probabilistic dynamic programming technique.
   c) Describe the algorithm of solving an integer programming problem. **(5+4+4)**

---

b) Define Lebesgue-Steiltjes measure. Determine the Lebesgue-Steiltjes measure on

i) set of discontinuity points

ii) $[0\ \infty)$ of a given non decreasing right continues function F(X).

$$F(X)=\begin{cases} 0 & \text{if } x<0 \\ \dfrac{1+x}{4} & \text{if } 0\le x<1 \\ \dfrac{3x^2+29}{64} & \text{if } 1\le x<2 \\ 1-\dfrac{1}{8x} & \text{if } x\ge 2 \end{cases}$$

c) If A and B are independent events then show that

i) $(A \cap B^c)$ and

ii) $(A^c \cap B)$ also independent. **(6+4+3)**

4. a) State Jordan decomposition theorem. Obtain the Jordan decomposition of the following distribution function.

$$F(X)=\begin{cases} 0 & \text{if } x<-1 \\ \dfrac{2+x}{4} & \text{if } -1\le x<0 \\ \dfrac{2}{3} & \text{if } 0\le x<1 \\ 1 & \text{if } x\ge 1 \end{cases}$$

b) Define the distribution function of a vector random variable. State its properties. Illustrate for the bivariate case that it is non-decreasing with respect each of its arguments.

c) Check whether X^2 is random variable if X is a random variable in probability space. **(6+4+3)**

5. a) State and prove monotone convergence theorem.

b) Write a note on expectation of arbitrary random variable.

c) Prove that convergence in law implies convergence in probability if limiting random variable is a constant. **(6+4+3)**

Reg. No. [| | | | | | | |]

STH 403

First Semester M.Sc. Degree Examination, May 2022

STATISTICS

Probability Theory

Time : 3 Hours

Max. Marks : 70

***Note** : Question No. 1 is **compulsory**. Answer **any four** questions from the remaining **seven** questions.*

(3×6=18)

1. Answer **any six** sub divisions :

a) Define $\lim_n \inf A_n$ and $\lim_n \sup A_n$ for a sequence $\{A_n\ n \geq 1\}$ of subsets of Ω.

b) Examine whether the set of real numbers between (4, 5) is countable.

c) If X and Y are measurable function, then show that X + Y is also measurable.

d) Define Lebesgue measure. Prove that Lebesgue measure on a countable set is zero.

e) State Fatou's lemma.

f) Let $\{Y_n\}$ be a sequence of random variable with $P(Y_n = 0) = 1 - \frac{1}{n^2}$ and $P(Y_n = \pm n) = \frac{1}{2n^2}$; $n \geq 1$. Examine whether Central Limit Theorem (CLT) holds.

g) Derive the characteristic function of Bernoulli random variable.

h) Distinguish between Weak Law of Large Number (WLLN) and Strong Law of Large Number (SLLN). State Chebyshev's WLLN.

2. a) Show that the set i) (a, b) and ii) [a, b] where a, b ∈ R are Borel sets.

b) Obtain the limit of $\{A_n\}_{n=1}^{\infty}$ when $A_n = \left(-\frac{1}{2n}\ 0\right]$ if n is odd and $A_n = \left[0\ \frac{1}{2n}\right)$ if n is even.

c) Prove that every σ-field is monotone field and every monotone field is a σ-field.

(6+4+3)

3. a) State and prove the continuity theorem on measure.

P.T.O.

6. a) State and prove Borel-Cantelli lemma.

   b) Define convergence in r^{th} mean for a sequence of random variable $\{X_n\}$. Prove that convergence in r^{th} mean implies convergence in probability. **(8+5)**

7. a) If $X_n \xrightarrow{P} X$ then prove that $X_n \xrightarrow{L} X$ and state when the converse holds.

   b) Define Characteristic function. State and prove any two of its properties.

   c) Let X_n takes value $0, 1+\frac{1}{n}$ and $2+\frac{1}{n}$ with probability $\frac{1}{4}-\frac{1}{n}, \frac{1}{2}$ and $\frac{1}{4}+\frac{1}{n}$ respectively for n = 1, 2, ... Check whether $\{X_n\}$ converges in Law. **(6+4+3)**

8. a) Establish Lindeberg-Levy CLT.

   b) Examine whether WLLN holds for $\{X_n\}$ with

   $$(X_n = \pm 2^n) = \frac{1}{2^{n+1}}, P(X_n = \pm 1) = \frac{1}{2}\left(1-\frac{1}{2n}\right).$$

   c) State Kolmogorov's Generalised WLLN. **(6+4+3)**
